Ronald Russell Loose

DEVELOPMENT OF A NAVY SATELLITE NAVIGATION SYSTEM

DEVELOPMENT OF A
NAVY SATELLITE NAVIGATION SYSTEM

A Thesis

Presented in Partial Fulfillment of the Requirements
for the Degree Master of Science

by

Ronald Russell Loose, B.A.

The Ohio State University
1966

ACKNOWLEDGMENT

The author wishes to express his appreciation to Dr. Simo Laurila, Department of Geodetic Science at the Ohio State University, for his assistance and guidance during the preparation of this thesis.

Special acknowledgment is given to Dr. Mark M. Macomber, Commander, United States Navy and former Project Transit/Anna Officer, Bureau of Naval Weapons, for his assistance in the theoretical development of Chapter three. Dr. Macomber also offered very useful suggestions and criticism on the first draft of the manuscript.

I am indebted to the United States Naval Oceanographic Office, through whose facilities the greater portion of my reference material was made available.

Finally, I desire to acknowledge my indebtedness to the United States Navy, which has sponsered my studies in Geodetic Science.

TABLE OF CONTENTS

TABLE OF CONTENTS
(continued)

LIST OF FIGURES

Figure | Page

LIST OF TABLES

[illegible]

[illegible]

[illegible]

1. INTRODUCTION

1.1 GENERAL. - - Marine navigation is the process of directing the movements of a vessel from one point to another. Historically, piloting, dead reckoning, and celestial methods have been used for this purpose. The earliest form of marine navigation was piloting, which came into being when man guided his vessel with respect to familiar landmarks. Dead reckoning probably followed as he attempted to predict his future positions in venturing beyond sight of land. While it is generally accepted that celestial bodies were used to steer by from almost the beginning, celestial navigation, as it is known today, had to wait until information regarding the motion of celestial bodies was acquired.[1]

The advent of World War II brought about the development of various navigational aids and electronic positioning systems which enabled the navigator to fix his position without regard to cloud cover or weather conditions. Since this time the trend in navigation equipment has been toward obtaining greater positional accuracy, continous position-fixing capabilities, and longer range operations. These improvements are being achieved by the use of electromagnetic and acoustic signals, sophisticated electronic and celestial methods, and inertial systems. Automatic computation has

[illegible]

become a necessity in the more sophisticated systems which require long and tedious calculations.

The navigation requirements of the Navy are quite different from those of the ordinary user. An ideal system, other than a completely self contained inertial system, would provide world wide coverage, all weather operation, an unlimited number of users, availability of frequent fixes, immunity to interference, passive user operation, and high positional accuracy. Certain military operations require positional accuracies of 0.1 nautical mile or better.

Present long range navigation systems and their characteristics are presented in Table 1.1,[2] where long range is defined as having a position fixing capability at a range of approximately 400 nautical miles or greater. Of these, the inertial and acoustic doppler systems are essentially sophisticated dead reckoning systems with potential world coverage and position indicating capability. However, at the present time and state of the art, the use of a navigational satellite system appears as the best solution to the requirements outlined above.

1.2 CONCEPTION OF A DOPPLER SATELLITE NAVIGATION SYSTEM. The concept of a doppler navigational satellite system was a by-product of the observation of the Soviet Union's first artificial earth satellite, Sputnik I, in the autumn of

TABLE 1.1 - LONG-RANGE NAVIGATION SYSTEMS

System	Estimated Useful Range	Estimated Accuracy	Frequency Range
Celestial	World-wide	3nm	-
Star Tracker	World-wide	0.3nm	-
Consol	700-1200nm	0.3-0.6°	190-194kc 257-263kc
RDF	300-600nm	2-6°	200-415kc 285-325kc
Decca	200-300nm	0.25-1.0nm	90-130kc
Loran-A	700-900nm	0.5-3.0nm	1.75-2mc
Loran-C	1200-1500nm	0.2-0.5nm	90-110kc
Omega	5000nm	0.5-1.0nm	10-15kc
Satellite	World-wide	0.1-2.0nm	100-400mc
Inertial	3-12 hours	2.0nm/hr	-
Acoustic Doppler	300 ft water depth	1% of velocity 1% of distance traveled	0.2-1mc

TABLE 1.1 (Cont)

System	Comments
Celestial	Inexpensive equipment, easy to use; limited to 2 fixes/day, affected by clouds and bad weather which also limits fix availability to 50%.
Star Tracker	Same as above except equipment very expensive.
Consol	Requires only a narrow band receiver to establish LOP.
RDF	World-wide coverage. U.S. coastal beacons. Capable of coarse fix only.
Decca	Easy to use, insufficient coverage, limited to about 240 mile range at night because of sky wave interference.
Loran-A	Easy to use, insufficient coverage, subject to sky wave interference. Less accuracy with sky wave.
Loran-C	Easy to use, accurate, insufficient coverage. Less accuracy with sky wave. Expensive installation.
Omega	World-wide coverage. Presently in advanced development. Will use three frequencies to yield both coarse and fine grids.
Satellite	World-wide coverage. Less sophisticated systems for comercial use now under study.
Inertial	Self contained, must be checked with outside reference. Accuracy decreases in a few hours at rapid rate. Very expensive, requires expert maintenance.
Acoustic Doppler	Sophisticated high accuracy systems utilize gyro-compass, velocity input, and automatic track-plotting equipment. Limited to shallow water.

1957. Dr. W. H. Guier and Dr. G. C. Weiffenbach of the Applied Physics Laboratory of Johns Hopkins University listened to the signal generated by Sputnik and noted the doppler shift in the transmitted radio frequency. The shift is given quantitatively by:

$$(1.1) \qquad \Delta f = -(\dot{\rho}/c)f_s$$

where Δf is the doppler shift from the satellite transmission frequency, $\dot{\rho}$ is the range rate, c is the velocity of electromagnetic propagation, and f_s is the satellite transmission frequency.[3] Hence, the measuring of doppler shift is comparable to measuring the range rate. This is the basis for the development of position determination equations derived in Chapter 3.

Guier and Weiffenbach deduced, and later demonstrated, that accurate measurement of the doppler shift pattern would permit the determination of a satellite orbit. After the first analysis and use of the doppler tracking technique, Dr. F. T. McClure of the Applied Physics Laboratory, reasoned that the technique could be inverted and that the doppler curve received from a satellite whose position was known could be used to determine the position of the receiver on the surface of the earth. It seemed only logical to utilize this deduction as a navigation system.

1.3 THE OPERATIONAL REQUIREMENT. - - The Navy is constantly seeking improvements in the fundamental accuracy

of navigation and in the operational character of navigation systems. Realizing that the doppler technique provides a powerful tool for simple but very accurate navigation, the development of a highly accurate operational navigation system using artificial satellites was made one of the Navy's first space objectives. The Chief of Naval Operations established an operational requirement for a navigational satellite as follows:

> Develop a satellite system to provide accurate, all-weather, world-wide navigation for naval surface ships, aircraft, and submarines.[4]

This requirement was generated by the need for a highly accurate position determination required by modern weapons systems. The satellite system was not developed to replace any of the present systems of navigation. Instead, it was developed to provide another navigation aid for determining positions of extremely high accuracy anywhere on the surface of the earth.

Accordingly, the Navy proposed to the Advanced Research Projects Agency (ARPA) that a project be undertaken to exploit the doppler technique for navigational purposes. The program began in 1958 as a feasibility study under contract with the Applied Physics Laboratory. This program was designated "Project Transit".

2. DEVELOPMENT OF AN OPERATIONAL SYSTEM

2.1 GENERAL CONFIGURATION AND OPERATION - - After the initial conception it did not take long to determine the configuration of an operational doppler satellite navigation system. The basic system consists of the following five groups of equipment:

1. Satellites.
2. Tracking stations.
3. Computing center.
4. Injection station.
5. Navigational equipment.

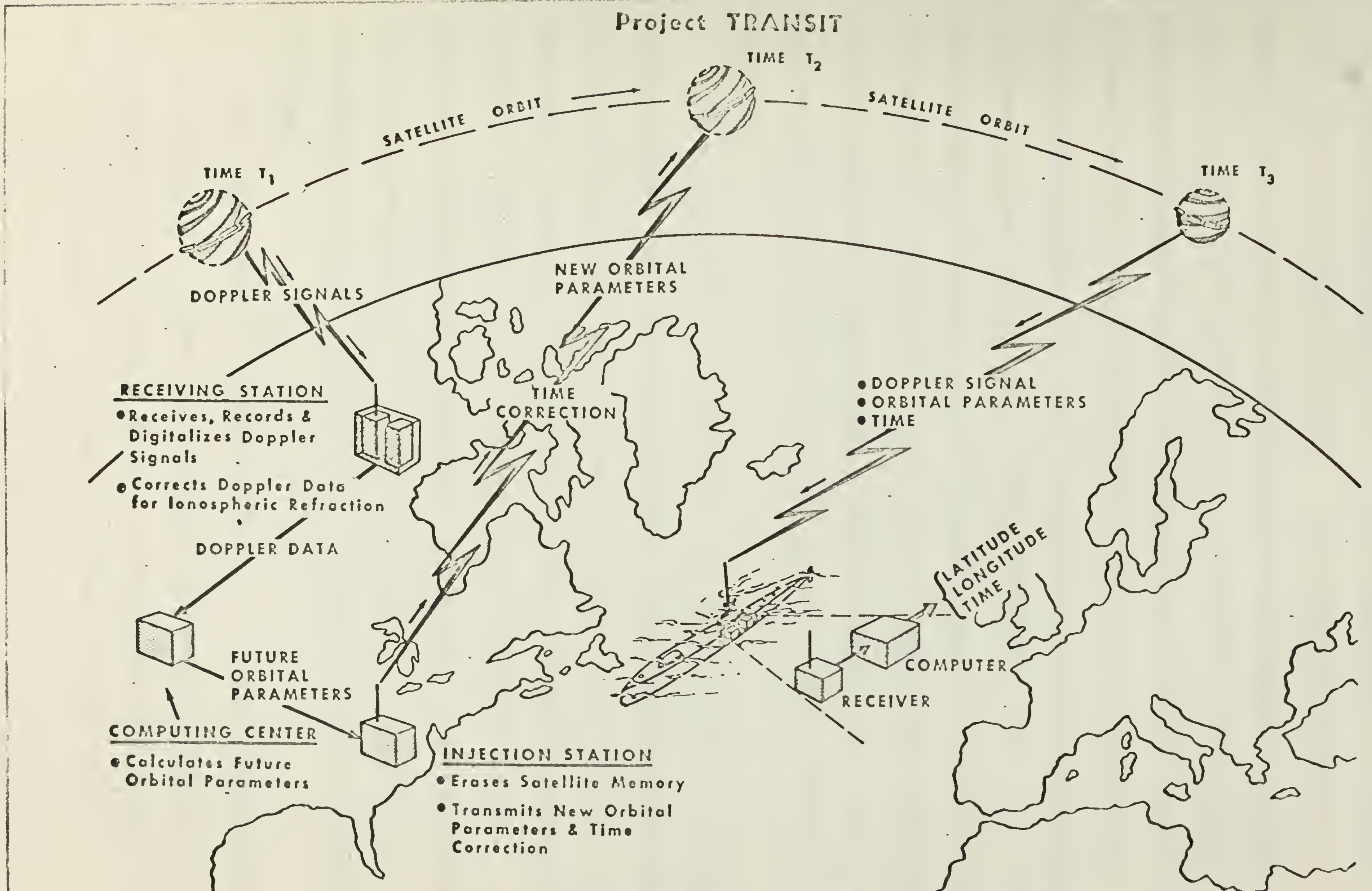
The system is designed to employ four satellites in polar or near polar orbits at an altitude of 600 miles and a period of 110 minutes. With orbital planes separated by 45 degrees a fix is available at least every 110 minutes. If the satellite passes within 10 degrees of the zenith, the interval is extended to 220 minutes since longitude can not be accurately determined in this situation. At higher latitudes the frequency of fix availability increases considerably. Typical satellite orbits for an operational system are illustrated in Figure 2.1.[5] The satellite transmits two rigorously coherent frequencies controlled from the same oscillator. This allows a first order correction to be made for ionospheric refraction. In

FIGURE 2.1 - TYPICAL OPERATIONAL SATELLITE ORBITS

addition the satellite transmits orbital information and time signals synchronized with UT-2_c. The time signals are obtained from a clock which is regulated by the satellite oscillator.

The overall plan of a typical system operation is illustrated schematically in Figure 2.2.[4] A navigational satellite is shown at three different time periods during an orbit. During time period T_1 the satellite signals are received by a ground tracking station as the satellite passes within radio line of sight. As each tracking station receives the transmitted signal, it determines the apparent doppler shift and reads out the satellite memory and time marker. These data are then transmitted in digital form by teletype to the computer center.

After receiving a sufficient number of pass reports, the computer center uses the doppler data to compute an improved satellite orbit and then predicts the satellite position for every two minutes in the next 16 hour period. When this is completed, the computer generates the parameters of the orbit from which the position of the satellite at any point in time can be determined. The computer also analyzes the time errors to give the clock rate, and determines the correction to both the clock setting and the clock rate. All of this information is then sent to the injection station.



SCHEMATIC SYSTEM OPERATION
FIGURE 2.2

During time period T_2, when the satellite is within range of the injection station, the satellite memory is erased, the new data is inserted into the memory, and the clock is reset and regulated. The satellite immediately retransmits this message to the injection station where it is compared with the information originally transmitted. If the readback indicates that the injection was unsuccessful, the message is automatically retransmitted to the satellite. When accurate storage in the satellite memory is verified, the memory is locked by a time clock. Additional information is rejected until approximately twelve hours later when the satellite is again in optimum range of the injection station.

After receiving the message from the injection station, the satellite continuously transmits the parameters of its orbit and its predicted orbital position at two minute intervals. During time period T_3 the navigation equipment aboard a ship receives the orbital parameters from the satellite and records the doppler shift for at least three 2 minute time intervals. Computing equipment uses this information to calculate the ship's true latitude, longitude and time.

2.2 FEASIBILITY DETERMINATION. - - The first phase of the Transit Program was primarily concerned with proving the feasibility of such a doppler satellite navigation system. This phase was under the direction of ARPA.

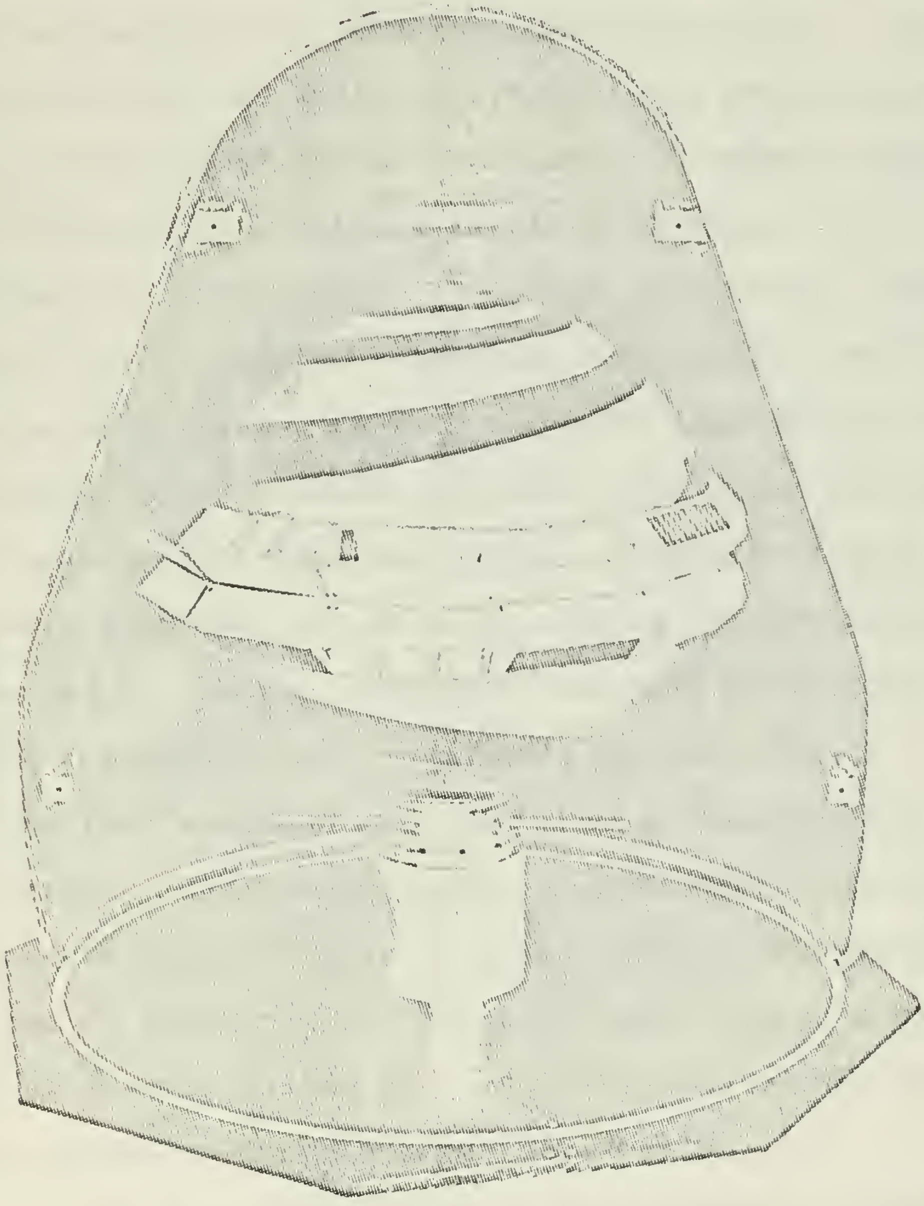
During this period four experimental satellites were designed. They were designated the 1A, 1B, 2A, and 2B respectively. Seven ground tracking stations were put into operation and others were in the process of being designed and constructed. An experimental computation center was placed into operation and a time and frequency standards laboratory was established at the Applied Physics Laboratory. The requirements of the injection station were determined and the design of the station was undertaken. Extensive mathematical analyses of computational procedures, ionospheric refraction, geodesy, orbit determination, and position determination were initiated.

On 17 September 1959 the Transit 1A satellite was launched by means of a Thor-Able vehicle. The satellite in it's flight shroud is shown in Figure 2.3.[4] Although the satellite failed to obtain orbit, sufficient data were obtained to verify the feasibility of satellite tracking and navigation by doppler analysis. However, objectives concerned with the verification of the satellite design were not achieved.

The Transit 1B satellite was similar to the 1A in appearance and function. The satellite consists of a shell divided into two hemispheres 36 inches in diameter, a central support, and an instrument tray. The shell is a lamination of fiberglas with a honeycomb plastic filler.

[illegible]

Project TRANSIT



TRANSIT 1A IN FLIGHT SHROUD

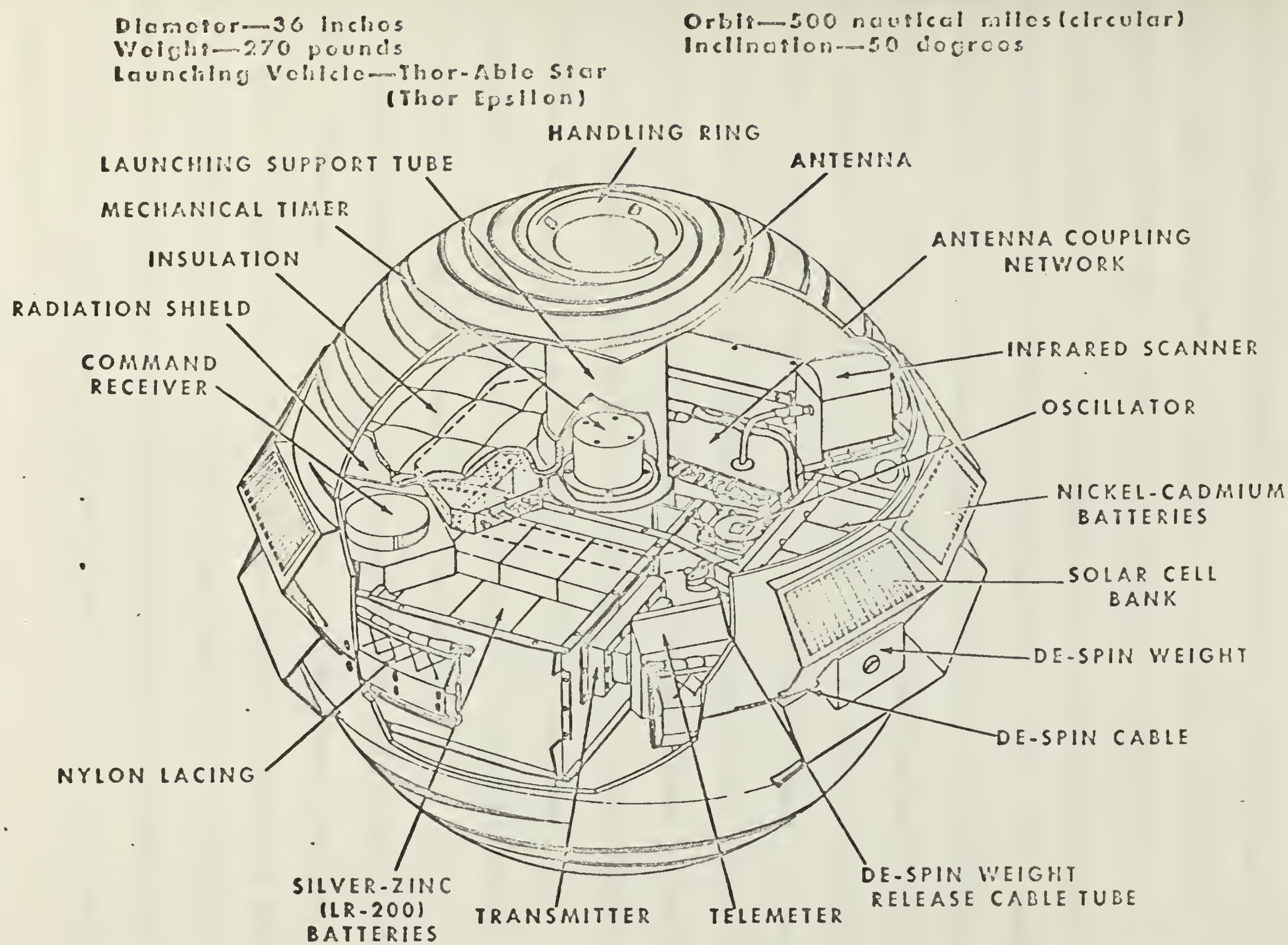
FIGURE 2.3

This provides a strong, highly insulated, nonmetallic structure. White paint on the shell surface aids in temperature control. The central support, also made of laminated fiberglas, connects the hemispheres of the shell and supports the aluminum instrument tray. The tray is tied to the shell by nylon lacing which absorbs the shocks produced during launching. A cutaway view of the Transit 1B satellite is given in Figure 2.4.

The tray contains two transmitting systems, silver-zinc and nickel-cadmium batteries, two command receivers, and a de-spin system. A radiation and insulation shield is placed above and below the tray. Two banks of solar cells used for charging the nickel-cadmium batteries and two de-spin weights are mounted outside and around the equator of the shell. Broadband antennas in a spiral pattern are painted on the outside of the shell.

The tray also contains an infrared scanner which was designed for special studies by the Naval Ordnance Test Center. A spin rate of approximately 180 rpm was imparted to the satellite for this test. After a pre-set time the de-spin system stops the spinning of the satellite, since spinning renders doppler data useless.

The Transit 1B satellite was successfully launched on 13 April 1960 by means of a two stage Thor-Able-Star vehicle. The elliptical orbit has an inclination of 51.3 degrees, a perigee of 235 miles, and an apogee of 475 miles.



TRANSIT IB
SATELLITE CUTAWAY VIEW

Rev APR 1960

FIGURE 2.4

The satellite period is 95.5 minutes. Although the satellite was not injected into the planned circular, 500 mile orbit, the ground tracking and computation proved completely successful. Analysis of telemetered data indicated that the satellite components functioned satisfactorily. All electrical and mechanical aspects of the satellite were proven including the solar cell operation, command off-on operation, de-spin devices, and temperature stability. Navigational experiments were conducted which gave position determinations to within $\frac{1}{2}$ mile of first order geodetic control points.[4] All calculations used in orbit and navigational fix determination employed the first order ionospheric refraction correction obtained from comparing the two coherent transmitted frequencies. The accuracy obtained indicated the validity of this two frequency technique. Investigations of the earth's gravitational field and geodetic surface were also conducted.

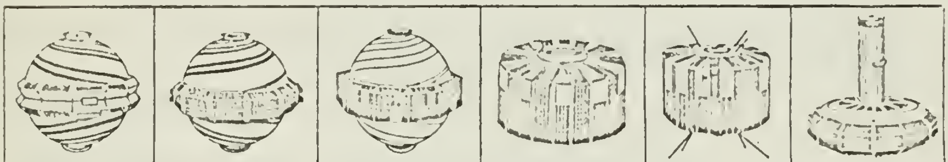
The Transit 1B satellite ceased radiating on 11 July 1960. However, the results of this experiment achieved the goal of the first phase of the Transit research program by conclusively proving the system feasibility while demonstrating the ability to perform doppler satellite navigation with an accuracy required for most military applications.

2.3 PROTOTYPE DEVELOPMENT. - - With the success of the Transit 1B satellite the Transit Program entered

the second phase of development in early May 1960 when the management of the Program was formally transferred from ARPA to the Bureau of Naval Weapons. This phase consisted of the development of an engineering prototype of the operational satellite navigation system. Consequently, a series of research and development satellites were constructed and placed into orbit. The basic characteristics and orbital information of these satellites is given in Table 2.1.[5] Effort was continued in improvement of system design, simplification, and reliability. Comprehensive testing in environmental chambers, and for vibration, shock, and acceleration were conducted on the satellite and it's components. A complete data storage and readout system was developed including the design and construction of an injection station. Tracking procedures and equipment were improved along with the development of shipboard navigation equipment. Continued research was conducted in the areas of gravity, refraction, and geodesy resulting in improved accuracy.

The Transit 2A satellite was placed into orbit on 21 June 1960. It differed from the 1A and 1B in that it had more solar cells, an improved telemetering system, and an electronic clock. The clock, running from the satellite oscillator, provided the accurate time correlation between the tracking stations which is required for accurate determination of satellite orbits.

TABLE 2.1 - TRANSIT RESEARCH AND DEVELOPMENT SATELLITES LAUNCHED DURING 1960 AND 1961



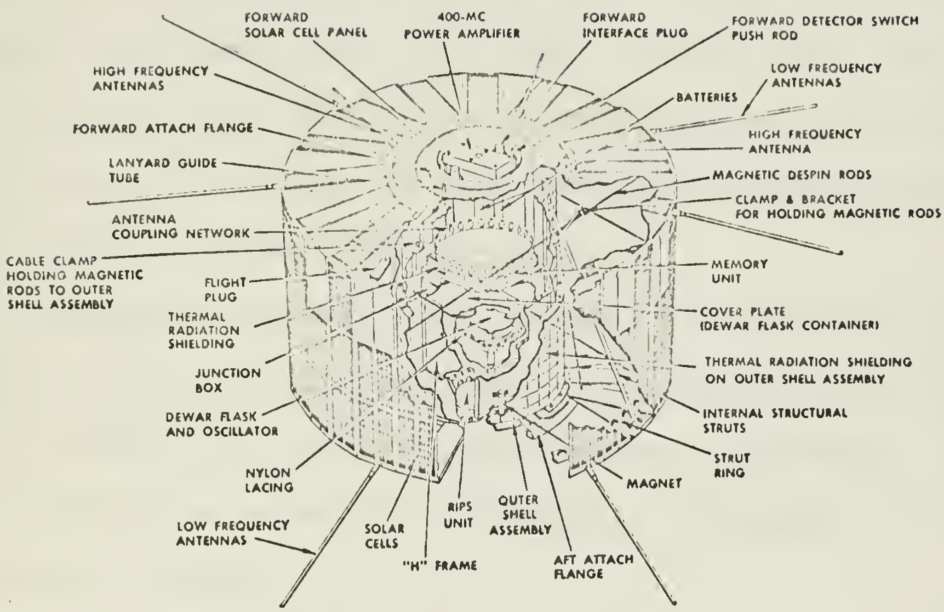
	Nav 1B	Nav 2A	Nav 3B	Nav 4A	Nav 4B	TRAAC
LAUNCH DATE:	APRIL 13, 1960*	JUNE 21, 1960	FEB 21, 1961**	JUNE 29, 1961	NOV 15, 1961	NOV 15, 1961***
FREQUENCIES "B" SYSTEM:	162-216mc	162-216mc	162-216mc			
"C" SYSTEM:	54-324mc	54-324mc	54-324mc	54-324mc	54-324mc	54-324mc
"Z" SYSTEM:				150-400mc	150-400mc	
RMS FREQUENCY NOISE AS MEASURED AT TRACKING STATIONS: "B" SYS. 162-216mc	5 Parts In 10^9	1 Part In 10^9	5 Parts In 10^{10}			
"C" SYS. 54-324mc	5 Parts In 10^{10}	7 Parts In 10^{10}	5 Parts In 10^{10}	2 Parts In 10^{10}		
"Z" SYS. 150-400mc				2 Parts In 10^{10}		
MEMORY: TYPE	None	None	Magnetic Shift Register	Delay Line	Ferrite Core	None
: CAPACITY			384 Bits	2049 Bits	1344 Bits	
WEIGHT	265 lbs.	223.3 lbs.	290.3 lbs.	175.1 lbs.	198 lbs.	233 lbs.
INITIAL PERIGEE ALTITUDE:	378 km	621 km	178 km	883 km	950 km	
INITIAL APOGEE ALTITUDE:	754 km	1070 km	978 km	994 km	1111 km	
INCLINATION:	51.3 deg.	66.7 deg.	28.4 deg.	66.8 deg.	32.4 deg.	32.4 deg.

* Note: Nav 1B ceased radiating on July 11, 1960.
** Nav 3B entered the atmosphere on March 30, 1961.
*** Launched pickaback on Transit 4B.

The Transit 3B satellite, launched on 21 February 1961, incorporated an additional electronic unit in the form of a complete data storage system capable of storing a small number of bits of digital information. A storage loading technique and readout system was used to insure correct data insertion. This provided the first flight test of the data system including the injection station. Unfortunately, the 3B satellite only remained in orbit a little over five weeks and the experimental results were quite limited.

The Transit 4A and 4B satellites were launched on 29 June 1961 and 15 November 1961 respectively. A cutaway view of the Transit 4A satellite is given in Figure 2.5.[5] The 4 series was extensively engineered to achieve an extremely stable oscillator, small size, and increased reliability. Crystal oscillators of high accuracy were carefully aged and selected. Their operational temperature was actively controlled by a small heating coil contained in the thermal inertial element. This emphasis on constant operating temperature yielded oscillator stabilities up to one part in 10^{10} for short intervals. Such accuracy gives the satellite system a capability for extremely precise position determination.

The internal equipment was miniaturized to reduce the satellite size and weight. This was to enable the



TRANSIT 4A

SATELLITE CUTAWAY VIEW

FIGURE 2.5

use of an inexpensive Scout launch vehicle. The outer shell of solar cells is collapsable and folds back against the satellite outer shell assembly. This greatly reduces the size of the satellite as shown in Figure 2.5. Once the satellite was put into orbit the outer shell of solar cells was to be mechanically extended. In practice this was never achieved since the launch vehicle was always large enough to allow the satellite to be fully extended within it's shroud. The 4 series also contained an experimental radio isotope (RIPS) power source.

In order to increase the satellite reliability, plated circuits and connection welding techniques were used throughout. In addition, entire sybsystems were imbedded in plastic to introduce the necessary ruggedness, strength, and temperature inertia.

All the experimental satellites so far have had to use isotropic antennas because they were not stabilized with respect to the earth. Since no antenna is truly isotropic the signal in some directions is much weaker than in others. As the satellite rotates, it sometimes sweeps a direction of low signal strength across the ground receiver and the signal lock can not be maintained. This accounts for almost all of the satellite passages that can not be used.

The Transit Research and Attitude Control (TRAAC) satellite was launched pickaback by the same vehicle as

the 4B satellite. Its primary objective was testing a method of gravity attitude stabilization. It uses the gradient of the gravitational field across the dimensions of the satellite in order to keep one face of the satellite always toward the earth. This allows the use of a directional antenna and avoids the problem of low signal strength in the antenna pattern.

As a result of the data gathered from the research and development satellites the basic characteristics and performance of a satellite navigation system were validated and a final prototype operational satellite was developed. The first of the operational Transit 5 series was launched in 1962. An artist's conception of the Transit 5A satellite is illustrated in Figure 2.6.[5] Details pertaining to the operational satellite and system are not readily available due to security restrictions. However, this satellite incorporates the design features proven in the previous experimental satellites. Conspicious additions include solar cell paddles and a damping system. The solar cell paddles can be angularly oriented with respect to the sun and thereby insure a nearly constant output of power. The damping system is composed of a long boom with a small weight attached to a spring at the end of the boom. The large displacements of the spring provides the necessary damping of angular oscillations required to keep the

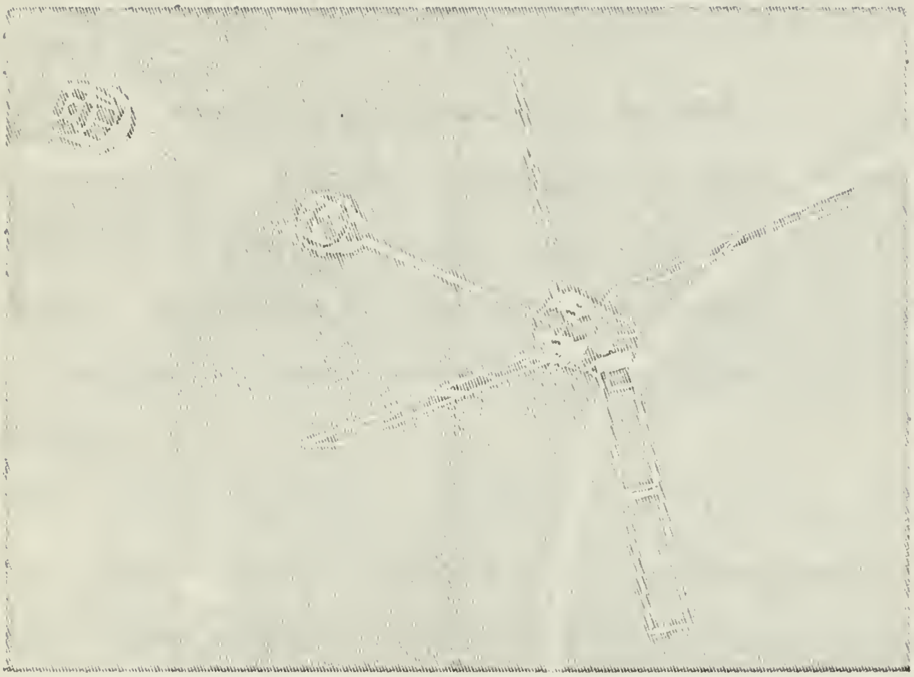


FIGURE 2.6 - PROTOTYPE OPERATIONAL SATELLITE
TYPE 5A

satellite gravity stabilized.

Since July 1962 the U. S. Naval Observatory has maintained equipment for the recovery of time signals from Transit type satellites to study the problems associated with providing continuous synchronization of UT-2_c. Precise time is required for highly accurate orbit and position determination.

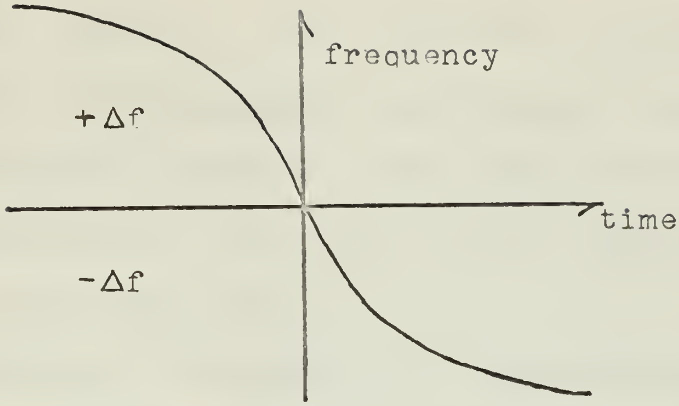
A later model of the prototype operational satellite was designed to use nuclear energy as a main power source. When this satellite was placed into orbit it was the first operational satellite to use this new energy in space. To date, a number of prototype satellites have been placed into orbit.

2.4 THE OPERATIONAL SYSTEM. - - In July of 1964 three of the prototype satellites were incorporated into an operational system. A complete operational ground network was set up and is entirely separate from the research and development network. The research and development network is still in existance and is designed to provide basic information on the size and shape of the earth and it's gravity field. This information is periodically processed through a computer at the Applied Physics Laboratory and occasionally new force terms are derived. The new terms are then used by the operational system to improve orbital updating and predictions. In July 1965 the name of the Transit System was officially changed to the Navy Navigation

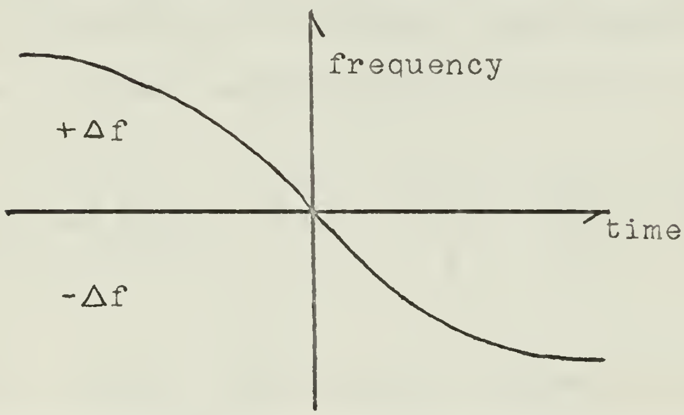
Satellite System. The use of Transit is no longer authorized within the Navy. A discussion of the operational system and it's components is presented in Chapter 4, following an investigation of the mathematical theory of satellite navigation presented in Chapter 3.

3. NAVIGATIONAL SATELLITE THEORY

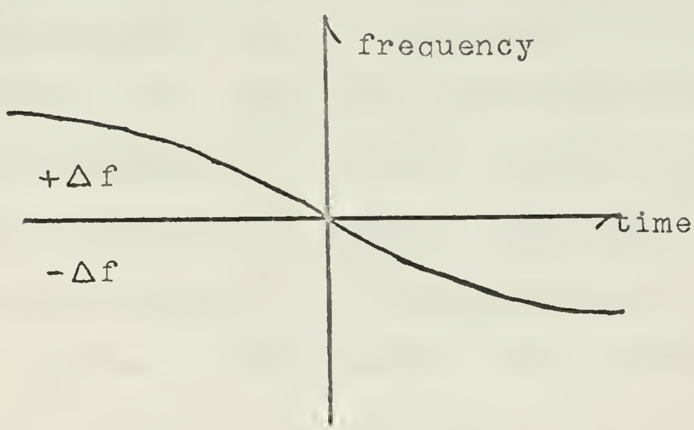
3.1 GENERAL. - - We will assume an artificial satellite in a known polar or near polar orbit transmitting on two extremely stable coherent frequencies in the 100 to 400 megacycle range with added uniformly spaced time signals. At a receiving station an apparent change in the satellite transmission frequency is observed. This is caused by the motion of the satellite and the receiving station. The amount is proportional to the velocity of approach or recession. The exact amount of this shift depends on the exact location of the receiving station with respect to the path of the satellite. This is illustrated in Figure 3.1. The navigation equipment integrates the heterodyne frequency, which is in the tens of kilocycles range, between the satellite transmission and a reference oscillator in the navigation equipment. We note from Figure 3.1 that the sign of the frequency shift changes from plus to minus at the center of passage. The beat note does not distinguish the sign of the frequency difference. In order to get correct measurements it is required that the beat frequency does not pass through zero. Therefore, the local oscillator frequency is offset from the satellite oscillator by 80 parts in 10^6. This offset exceeds the maximum doppler shift so that the beat note never changes sign. Upon receipt of each time signal the number of cycles counted during the previous



OBSERVER CLOSE TO SUBTRACK



OBSERVER AT INTERMEDIATE DISTANCE FROM SUBTRACK



OBSERVER DISTANT FROM SUBTRACK

FIGURE 3.1 - EFFECT OF OBSERVERS LONGITUDE DISPLACEMENT FROM SATELLITE SUBTRACK.

time interval is recorded. The navigation equipment uses the higher of the two frequencies to determine count and the comparison between the two for a first order correction for ionospheric refraction. It is then assumed that the recorded cycle count is refraction free.

3.2 REFRACTION CORRECTION. - - Accurate doppler measurements have utilized the dispersive nature of the ionosphere to eliminate the refraction contribution which is expressed as a series expansion in the inverse frequency.[5]

$$(3.1) \quad \Delta f(t,f_s) + \frac{f_s}{c}\dot{\rho}(t) = \frac{A(t)}{f_s} + \frac{B(t)}{f_s^2} + \frac{C(t)}{f_s^3} + \frac{D(t)}{f_s^4} + \cdots$$

By receiving two coherent frequencies transmitted from the satellite the frequencies may be substituted into (3.1) and the first unknown coefficient 'A' determined at each time value t , where $\dot{\rho}$ is the range rate and c the velocity of electromagnetic propagation. With the use of only two frequencies the resulting cycle count still contains all contributions that depend on the higher powers of $(1/f_s)$. In operational systems two frequencies are generally adequate for most precision calculations. The tracking station receiver uses analog methods to automatically make the refraction correction before the doppler data are digitized. A further discussion of ionospheric refraction is presented in Section 5.2.

Project TRANSIT

IONOSPHERIC REFRACTION

FIGURE 3.2

3.3 DETERMINATION OF THE INTEGRATED CYCLE COUNT. - - The satellite transmits a time signal at T_1 and at T_2. The transmission times from the satellite to the receiver are t_1 and t_2. If we denote:

f_s = satellite transmission frequency.

f_r = reference frequency.

f_s^* = observed satellite transmission frequency affected by doppler shift.

IC = integrated cycle count.

we can derive the following relationships.

The heterodyne frequency observed at the receiver is:

$$f_h = f_s^* - f_r \tag{3.2}$$

and the integrated cycle count is obtained from:

$$IC = \int_{T_1+t_1}^{T_2+t_2} (f_s^* - f_r)dt \tag{3.3}$$

Splitting the integral and evaluating the last term:

$$IC = \int_{T_1+t_1}^{T_2+t_2} (f_s^*)dt - f_r \left[(T_2+t_2) - (T_1+t_1) \right] \tag{3.4}$$

The integral of (3.4) is nothing more than the number of cycles transmitted by the satellite during the uniform time interval $T_2 - T_1$.

Therefore, we can say:

$$(3.5) \qquad \int_{T_1+t_1}^{T_2+t_2} (f_s^*)dt = \int_{T_1}^{T_2} (f_s)dt$$

Substituting the result of (3.5) into (3.4):

$$(3.6) \qquad IC = f_s(T_2 - T_1) - f_r\left[(T_2+t_2) - (T_1+t_1)\right]$$

By rearranging terms (3.6) can be written:

$$(3.7) \qquad IC = \Delta f \Delta T - f_r \Delta t$$

Where:

$$\Delta f = f_s - f_r$$

$$\Delta T = T_2 - T_1$$

$$\Delta t = t_2 - t_1$$

Integration allows the use of range differences (see Equation 3.9) instead of range rate in computations.

3.4 RELATION OF DISTANCE AND INTEGRATED CYCLE COUNT. - - The basic distance formula states that distance is equal to velocity times time. Hence, the difference in transmission times from the satellite is proportional to the difference in range at T_1 and T_2. Solving (3.7) for Δt:

$$(3.8) \qquad \Delta t = \frac{\Delta f \Delta T - IC}{f_r}$$

If we let r equal the range to the satellite we can express (3.8) in terms of distance.

$$(3.9) \qquad \Delta r = \frac{c(\Delta f \Delta T - IC)}{f_r}$$

With this basic equation relating distance and integrated cycle count we can derive equations for position determination in a rectangular coordinate system.

3.5 EQUATIONS FOR POSITION DETERMINATION. - -

If we denote:

r = true distance to the satellite.

R = computed distance to the satellite from a DR position.

X_n, Y_n, Z_n = assumed ship coordinates in a right hand orthogonal coordinate system. (obtained from some other navigation system)

X_i^s, Y_i^s, Z_i^s = satellite coordinates in the same system.

$i = 0,1,2,\ldots\ldots N$ - where N is equal to the number of time points.

Then:

$$(3.10) \qquad r = R + dR$$

Using assumed values for the variables as opposed to the true values in (3.9) and differentiating (3.9) and (3.10) with respect to the variables we arrive at:

$$(3.11) \qquad \Delta r = R_{i+1} - R_i + dR_{i+1} - dR_i = \frac{c}{f_r}(\Delta f_n \Delta T - IC_n) + \frac{c}{f_r}(\Delta T d \Delta f_n) - \frac{c}{f_r} dIC_n$$

Expressing the R and dR terms in terms of our orthogonal coordinate system:

$$(3.12) \qquad R_i = \left[(X_i^s - X_n - i\Delta x)^2 + (Y_i^s - Y_n - i\Delta y)^2 + (Z_i^s - Z_n - i\Delta z)^2\right]^{\frac{1}{2}}$$

$$(3.13) \qquad dR_i = -\frac{1}{R_i}\left[(X_i^s - X_n - i\Delta x)dX_n + (Y_i^s - Y_n - i\Delta y)dY_n + (Z_i^s - Z_n - i\Delta z)dZ_n\right]$$

Where $\Delta x, \Delta y$, and Δz account for navigator motion between observations. Similiar equations can be developed for R_{i+1} and dR_{i+1}. By substitution of values obtained above into (3.11) Δr can be expressed as:

$$(3.14) \quad \Delta r = R_{i+1} - R_i + \frac{1}{R_i}\left[(X_i^s - X_n - i\Delta x)dX_n + (Y_i^s - Y_n - i\Delta y)dY_n + (Z_i^s - Z_n - i\Delta z)dZ_n\right]$$
$$- \frac{1}{R_{i+1}}\left[(X_{i+1}^s - X_n - (i+1)\Delta x)dX_n + (Y_{i+1}^s - Y_n - (i+1)\Delta y)dY_n + (Z_{i+1}^s - Z_n - (i+1)\Delta z)dZ_n\right]$$

Using this expression for Δr in (3.11) and rearranging terms we can form an observation type equation where v is the residual formed by the adjusted value of Δr minus the observed value of Δr.

$$v = \Delta r_a - \Delta r_b$$

If we denote the coefficients of (3.14) as follows:

a_i = coefficient of dX_n

b_i = coefficient of dY_n

c_i = coefficient of dZ_n

the equation takes the form:

$$(3.15) \qquad v_i = a_i dX_n + b_i dY_n + c_i dZ_n - \frac{c}{f_r} \Delta T \, d\Delta f_n + (R_{i+1} - R_i) - \frac{c}{f_r} (\Delta f_b \Delta T - IC_b)$$

In this equation we assume our cycle count is errorless and therefore neglect the dIC term. The navigator may also assume that the frequency of his reference oscillator is unknown but constant during the short interval of observation. In the equation this is treated as the unknown $d\Delta f$. The above equation contains four unknowns (dX_n, dY_n, dZ_n, $d\Delta f_n$). If we have four satellite observations we can solve for the unknowns and thus locate our position.

Equation (3.15) has been called an observation type equation instead of an observation equation because the residual is formed in terms of Δr instead of IC. In Section 3.7 the actual observation equation is derived where v is the residual formed by the adjusted value of IC minus the observed value of IC. It will also be shown that for navigational purposes Equation (3.15) may be used as the actual observation equation in computations.

If we assume the earth to be approximated by an ellipsoid of revolution then:

$$(3.16) \qquad \frac{X_n^2 + Y_n^2}{a^2} + \frac{Z_n^2}{b^2} = 1$$

We can reduce the number of unknowns to three by expressing dZ_n in terms of dX_n and dY_n. Differentiating (3.16):

$$(3.17) \qquad dZ_n = \left(- \frac{X_n}{Z_n} dX_n - \frac{Y_n}{Z_n} dY_n \right) \left(1 - e^2 \right)$$

The unknowns are now dX_n, dY_n, and $d\Delta f$. Now only three observations of a satellite are required to fix a position. By substitution of the value of dZ_n from (3.17) into (3.14), multiplying and gathering terms, we can form an observation equation similar to (3.15).

$$(3.18) \qquad v_i = R_{i+1} - R_i + \left[\frac{1}{R_i} \left((X_i^s - X_n - i\Delta x) + (Z_i^s - Z_n - i\Delta z)(1-e^2)\left(-\frac{X_n}{Z_n}\right) \right) - \frac{1}{R_{i+1}} \left((X_{i+1}^s - X_n - (i+1)\Delta x) + (Z_{i+1}^s - Z_n - (i+1)\Delta z)(1-e^2)\left(-\frac{X_n}{Z_n}\right) \right) \right] dX_n +$$

$$\left[\frac{1}{R_i} \left((Y_i^s - Y_n - i\Delta y) + (Z_i^s - Z_n - i\Delta z)(1-e^2)\left(-\frac{Y_n}{Z_n}\right) \right) \right.$$

$$-\frac{1}{R_{i+1}}\Big[(Y^s_{i+1} - Y_n - (i+1)\Delta y) + (Z^s_{i+1} - Z_n - (i+1)\Delta z)$$

$$(1-e^2)(-\frac{Y_n}{Z_n})\Big] dY_n - \frac{c}{f_r}(\Delta f_b \Delta T - IC_b) - \frac{c}{f_r}\Delta T\, d\Delta f_n$$

This equation can be reduced to the form:

(3.19) $$v_i = a_i dX_n + b_i dY_n + c_i d\Delta f_n + L_i$$

where a_i = coefficient of dX_n

b_i = coefficient of dY_n

c_i = coefficient of $d\Delta f_n$

$L_i = (R_{i+1} - R_i) - \frac{c}{f_r}\Delta f_b \Delta T - IC_b$

In practice we constrain our position to be on a sphere since we are dealing with small quantities.

Equation (3.16) becomes:

(3.20) $$X_n^2 + Y_n^2 + Z_n^2 = R_e^2$$

and (3.17) becomes:

(3.21) $$dZ_n = -\frac{X_n}{Z_n} dX_n - \frac{Y_n}{Z_n} dY_n$$

The maximum difference in the dZ_n term resulting from this approximation can be determined as follows:

(3.22) $$dZ_{n\,(ellipsoid)} = (1 - e^2) dZ_{n\,(sphere)}$$

and

$$dZ_{n\,(ellipsoid)} - dZ_{n\,(sphere)} = - e^2 dZ_{n\,(sphere)}$$

Therefore, the difference in (3.17) due to constraint to a sphere is $-e^2 dZ_{n(\text{sphere})}$. The term dZ_n has it's greatest value at the equator. Assuming that the navigator position is known to an accuracy of one nautical mile ($\simeq$ 1829 meters) and using the satellite derived value of e^2 from the Kaula Ellipsoid of 1965, we find the maximum difference in dZ_n to be:

$$\Delta dZ_n \simeq (0.00669)(1829) \quad \text{or} \quad \simeq 12.2 \text{ meters.}$$

This result is considered negligible for the purpose of navigation. By using the spherical approximation the $(1 - e^2)$ terms in the coefficients a_i and b_i are eliminated and the observation equation retains the form of (3.19). This is the general satellite observation equation.

3.6 REDUCTION OF OBSERVATION EQUATIONS. - - Three observations of a satellite will yield three observation equations. In practice these are used directly in the determination of the three unknowns. When there are more observations than unknowns the method of least squares may be applied to find that system of values which is most probable from the observations themselves. Arbitrarily assigning a weight of unity to each observation and adding the square sums of the coefficients we obtain:

$$(3.23) \quad [vv] = [aa]\, dX_n^2 + 2\,[ab]\, dX_n dY_n + 2\,[ac]\, dX_n d\Delta f_n + 2\,[aL]\, dX_n +$$

$$[bb]\, dY_n^2 + 2\,[bc]\, dY_n d\Delta f_n + 2\,[bL]\, dY_n + [cc]\, d\Delta f_n^2 + 2\,[cL]\, d\Delta f_n + [LL]$$

where: $[aa] = a_i a_i + a_{i+1} a_{i+1} \;\cdots\cdots\; a_N a_N$

By the method of least squares the most likely solution is when the sum of the squares of the residuals $[vv]$ is a minimum. To obtain this condition we set the partial derivatives of (3.23) with respect to the three variables equal to zero. This results in the formation of three normal equations.

$$[aa]\, dX_n + [ab]\, dY_n + [ac]\, d\Delta f_n + [aL] = 0$$

(3.24) $$[ab]\, dX_n + [bb]\, dY_n + [bc]\, d\Delta f_n + [bL] = 0$$

$$[ac]\, dX_n + [bc]\, dY_n + [cc]\, d\Delta f_n + [cL] = 0$$

Equations (3.24) are normal equations reduced from satellite observation equations. Regardless of the number of observation equations we can always reduce them to a number of normal equations equal to the number of unknowns. The solution of the normal equations yields unique results for the unknowns dX_n, dY_n, and $d\Delta f_n$. Taking the values for dX_n and dY_n we can solve for dZ_n. Adding these corrections to the original values for X_n, Y_n, and Z_n gives the rectangular coordinates of our adjusted position.

(3.25) $$X_a = X_n + dX_n$$

$$Y_a = Y_n + dY_n$$

$$Z_a = Z_n + dZ_n$$

The value obtained for $d\Delta f_n$ may be used to to update the frequency of the reference oscillator in the navigation equipment.

It should be again noted that the use of the least squares solution is advoided in practice by using only three observations to solve directly for the unknowns. If more than three observations are obtained the excess observations may be neglected or they may be combined to produce a total number of three. This is the most practical solution since the advantages of the least squares solution are minimized by the small number of observations that are available during one satellite pass. During a maximum pass time of approximately 15 minutes only seven observations can be made. At the present time a decrease in the time interval to gain more observations is not practical due to computational time requirements and restrictions of the satellite memory and transmission system.

3.7 DERIVATION OF THE OBSERVATION EQUATION. - - In Section 3.5 an observation type equation was formed where v is the residual formed by the adjusted value of Δr minus the observed value of Δr. In the theoretical case the observation equation should be derived in terms of the

observed quantity. Such an observation equation will now be developed in terms of the integrated cycle count where the residual is formed by the adjusted IC minus the observed IC. It will then be shown that for the purpose of navigation the actual observation equation can be replaced by Equation (3.15).

Using the notation of Section 3.5 and denoting:

IC_a = adjusted integrated cycle count.

IC_b = observed integrated cycle count.

we can express IC as a function of the unknown parameters.

$$(3.26) \qquad IC_a = f(X_a,\ Y_a,\ Z_a,\ \triangle f_a)$$

and:

$$(3.27) \qquad IC_a = IC_b + v \quad \text{or} \quad v = IC_a - IC_b$$

The adjusted parameters can be expresses as:

$$X_a = X_n + x$$
$$Y_a = Y_n + y$$
$$Z_a = Z_n + z$$
$$\triangle f_a = \triangle f_n + F$$

where x, y, z, and F are corrections to the assumed navigator parameters. Assuming that the corrections are small compared to the parameters we can expand (3.27) using a Taylor's series where terms of the second order and greater are neglected.

$$(3.28) \quad IC_b + v = f(X_n, Y_n, Z_n, \Delta f_n) + \frac{\partial f}{\partial X_n} x + \frac{\partial f}{\partial Y_n} y + \frac{\partial f}{\partial Z_n} z + \frac{\partial f}{\partial \Delta f_n} F + \cdots\cdots$$

where:

$$f(X_n, Y_n, Z_n, \Delta f_n) = IC_n$$

The form of the actual observation equation for doppler satellite observations can be expressed as:

$$(3.29) \quad v_i = \frac{\partial f}{\partial X_n} x + \frac{\partial f}{\partial Y_n} y + \frac{\partial f}{\partial Z_n} z + \frac{\partial f}{\partial \Delta f_n} F + IC_n - IC_b$$

In order to evaluate the coefficients of the corrections we note from (3.9):

$$IC = \Delta T \Delta f - \frac{f_r}{c} \Delta r$$

and by substitution:

$$(3.30) \quad IC = \Delta T \Delta f_n - \frac{f_r}{c} \left[\left((X^s_{i+1} - X_n)^2 + (Y^s_{i+1} - Y_n)^2 + (Z^s_{i+1} - Z_n)^2 \right)^{\frac{1}{2}} - \left((X^s_i - X_n)^2 \quad (Y^s_i - Y_n)^2 \quad (Z^s_i - Z_n)^2 \right)^{\frac{1}{2}} \right]$$

The coefficients can now be determined from (3.30).

$$\frac{\partial IC}{\partial X_n} = -\frac{f_r}{c} \left[\left(-\frac{1}{R_{i+1}} (X^s_{i+1} - X_n) \right) + \left(\frac{1}{R_i} (X^s_i - X_n) \right) \right] dX_n$$

$$\frac{\partial IC}{\partial Y_n} = -\frac{f_r}{c}\left[\left(-\frac{1}{R_{i+1}}(Y^s_{i+1} - Y_n)\right)\left(\frac{1}{R_i}(Y^s_i - Y_n)\right)\right] dY_n$$

$$\frac{\partial IC}{\partial Z_n} = -\frac{f_r}{c}\left[\left(-\frac{1}{R_{i+1}}(Z^s_{i+1} - Z_n)\right)\left(\frac{1}{R_i}(Z^s_i - Z_n)\right)\right] dZ_n$$

$$\frac{\partial IC}{\partial \Delta f_n} = \Delta T \, d\Delta f_n$$

If we denote:

$a_i^* =$ coefficient of dX_n

$b_i^* =$ coefficient of dY_n

$c_i^* =$ coefficient of dZ_n

we can write (3.29) in the form:

$$(3.31) \qquad v_i = a_i^* dX_n + b_i^* dY_n + c_i^* dZ_n + \Delta T d\Delta f_n + IC_n - IC_b$$

If we multiply (3.31) by a constant equal to $-\frac{c}{f_r}$ we arrive at:

$$(3.32) \qquad -\frac{c}{f_r} v_i = a_i dX_n + b_i dY_n + c_i dZ_n - \frac{c}{f_r}\Delta T d\Delta f_n - \frac{c}{f_r}(IC_n - IC_b)$$

The coefficients of this equation are equal to the coefficients of (3.15). From (3.15) we obtain:

$$\Delta r_n = (R_{i+1} - R_i) = \frac{c}{f_r}(\Delta T \Delta f_n - IC_n)$$

and:

$$\Delta r_b = \frac{c}{f_r}(\Delta T \Delta f_b - IC_b)$$

(3.15) can now be expressed as:

$$(3.33)\quad v_i = a_i dX_n + b_i dY_n + c_i dZ_n - \frac{c}{f_r}\Delta T d\Delta f_n - \frac{c}{f_r}(IC_n - IC_b) + \frac{c}{f_r}\Delta T(\Delta f_n - \Delta f_b)$$

From Equations (3.32) and (3.33) the relation between the actual observation equation and the observation type equation based on terms of Δr can be expressed as:

$$(3.34)\qquad -\frac{c}{f_r}v_i^{IC} = v_i^{\Delta r} + \frac{c}{f_r}\Delta T(\Delta f_n - \Delta f_b)$$

and:

$$(3.35)\qquad v_i^{\Delta r} = Kv_i^{IC} + K\Delta T(\Delta f_n - \Delta f_b)$$

where K is a constant equal to $-\frac{c}{f_r}$.

Equation (3.35) shows that $v_i^{\Delta r}$ is related to v_i^{IC} by a constant except for the second term on the right side. It will now be shown that this second term can be neglected for the purpose of navigation. From (3.6) we note:

$$\Delta f = f_s - f_r$$

and:

$$\Delta f_n = f_s - f_{rn}$$

$$\Delta f_b = f_s - f_{rb}$$

From this it can be determined that the term $(\Delta f_n - \Delta f_b)$ is equal to $(f_{rb} - f_{rn})$. We will now determine the approximate maximum magnitude of the second term. Knowing that f_r has a drift rate of less than 2 parts in 10^{10} per day and assuming that we use the solution of $d\Delta f_n$ from previous observations to correct f_r, we can keep f_r to approximately 2 parts in 10^{10}. Therefore, the term $(\Delta f_n - \Delta f_b)$ has an approximate maximum value of $(\pm 2^2 + \pm 2^2)^{\frac{1}{2}}$ or ± 2.8 parts in 10^{10}. Using a value of 186,000 miles/sec for c and 400 mc/sec for f_r the approximate maximum value of the second term of (3.35) can be determined.

$$K\Delta T(\Delta f_n - \Delta f_b) \simeq \frac{186{,}000}{4 \times 10^8} (120) \frac{2.8}{10^{10}} (4 \times 10^8)$$

or $\simeq \pm 0.0062$ miles or ± 12 meters.

Since this value can be considered negligible for navigational purposes we can write:

(3.36) $$v_i^{\Delta r} = K v_i^{IC}$$

Therefore, if we set $\left[v_i^{IC} v_i^{IC} \right]$ to a minimum it follows that

$\left[v_i \triangle r \, v_i \triangle r\right]$ also goes to a minimum. Hence we are able to use (3.15) in place of (3.31) in our computations.

3.8 EARTH RECTANGULAR COORDINATES. - - The adjusted coordinate values in a rectangular system are of little practical use to the navigatior. Positional coordinates are desired in terms of latitude and longitude. While the calculations for coordinate determination and transformation are generally well known, they are included to provide a complete development of a position determination from doppler satellite observations.

In calculating the positions of navigational fixes on the surface of the earth it is necessary to consider these points as lying upon some mathematical surface. The actual shape of the earth's surface is quite irregular and is not adapted to the purpose of computation. For this reason it is necessary to adopt an assumed figure which will not depart from the true surface by an amount sufficient to produce serious errors in our results. The figure generally adopted is an ellipsoid of revolution which is defined by a given flattening (f) and a given semi-major axis (a).

The position derivation given in the preceeding sections is with respect to an arbitrary right hand orthogonal coordinate system. We will now define this system with respect to the ellipsoid of revolution assumed to represent the figure of the earth. The origin is at the center of the ellipsoid which is also the assumed center of mass. The

Z axis coincides with the mean rotational axis and the X axis intersects the Greenwich Meridian at the equator. The Y axis completes the right hand system. The coordinate system thus defined is illustrated in Figure 3.3.

The relations between geodetic latitude and longitude and the defined XYZ coordinate system, which can be derived from Figure 3.3, are given in the following formulas.

$$X = \frac{a \cos \varphi \cos \lambda}{W}$$

$$(3.37) \qquad Y = \frac{a \cos \varphi \sin \lambda}{W}$$

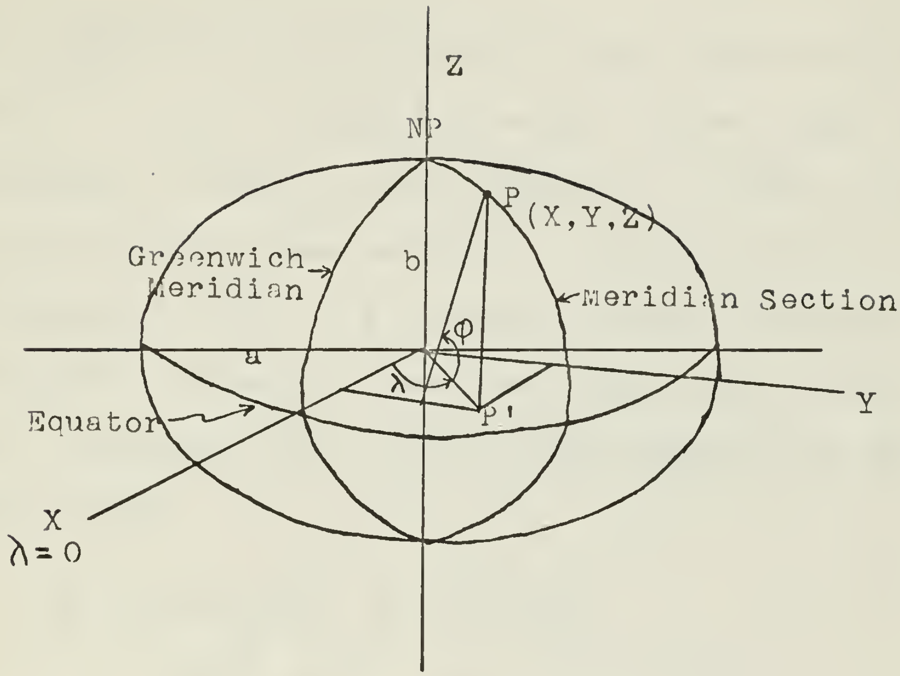
$$Z = \frac{a(1 - e^2) \sin \varphi}{W}$$

Where W is defined as:

$$W = (1 - e^2 \sin^2 \varphi)^{\frac{1}{2}}$$

3.9 DETERMINATION OF LATITUDE AND LONGITUDE. - - Using values for X_a, Y_a, and Z_a from (3.25) and the relations given in (3.37) we are able to determine the latitude and longitude on the surface of the given reference ellipsoid. The value for longitude can be solved directly from (3.37).

$$(3.38) \qquad \lambda = \mathrm{Tan}^{-1} \frac{Y_a}{X_a}$$



a = semi major axis

b = semi minor axis

e = first eccentricity $\frac{\sqrt{a^2 - b^2}}{a}$

φ = geodetic latitude

λ = geodetic longitude

FIGURE 3.3 - EARTH RECTANGULAR COORDINATE SYSTEM

The solution for latitude is a little more involved. From Figure 3.4 we observe that:

$$(3.39) \qquad \tan\varphi = \frac{N \sin\varphi}{(X^2\ Y^2)^{\frac{1}{2}}}$$

Where N is the radius of curvature of the surface of the ellipsoid in a plane through the normal and at right angles to the meridian. This is also referred to as the radius of curvature in the prime vertical. A problem arises that N is a function of φ. (N is equal to a/W) To solve (3.39) we must eliminate φ from the right side of the equation. Going back to (3.37) we can express Z in the following manner:

$$(3.40) \qquad Z = N \sin\varphi - N e^2 \sin\varphi$$

$$\text{and } N \sin\varphi = Z + N e^2 \sin\varphi$$

Substituting (3.40) into (3.39):

$$(3.41) \qquad \tan\varphi = \frac{Z + N\ e^2 \sin\varphi}{(X^2\ Y^2)^{\frac{1}{2}}} = \frac{Z}{(X^2\ Y^2)^{\frac{1}{2}}}\left(1 + \frac{N\ e^2 \sin\varphi}{Z}\right)$$

Substituting $N(1 - e^2) \sin\varphi$ for Z in the denominator of (3.41) we arrive at:

$$(3.42) \qquad \varphi = \mathrm{Tan}^{-1}\left(\frac{Z_a}{(X_a^2\ Y_a^2)^{\frac{1}{2}}}\right) \frac{1}{1 - e^2}$$

The values of latitude and longitude obtained from (3.38) and (3.42) are used by the navigator to fix his position

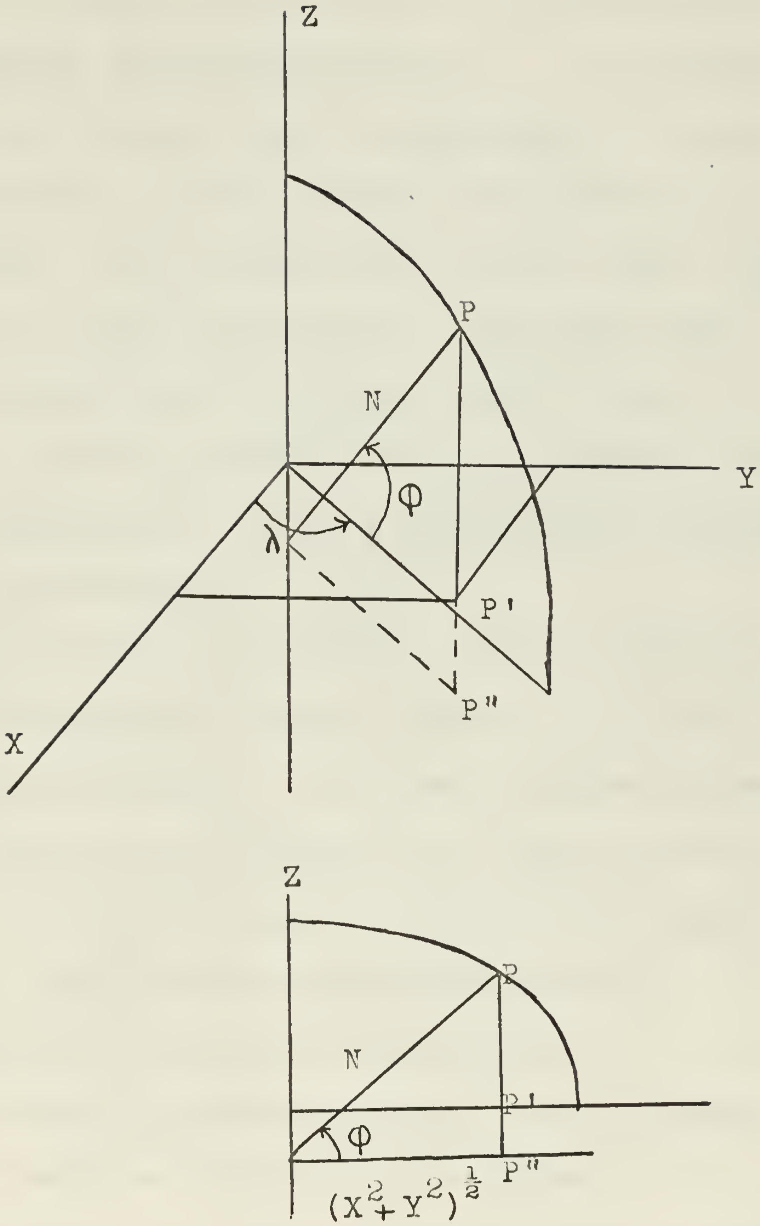


FIGURE 3.4 - MERIDIAN SECTION IN EARTH COORDINATE SYSTEM.

on the surface of the ellipsoid.

Reversing this procedure the navigator can easily calculate his position in earth rectangular coordinates from values of latitude and longitude by applying (3.37).

3.10 INERTIAL COORDINATE SYSTEM. - - The derivations to this point have assumed that the navigator is given satellite coordinates in the defined earth rectangular coordinate system. In practice the navigator must calculate these coordinates from given orbital parameters which give satellite coordinates in an inertial system. The determination of these parameters is quite complex and is omitted from this thesis. Readers desiring this information are referred to Reference 7.

The defined inertial coordinate system is illustrated in Figure 3.5. The origin of this system is the same as that of the earth rectangular coordinate system defined in Section 3.8. The Z_I axis coincides with the pole of the orbital plane. The X_I axis passes through the perigee and the Y_I axis completes the right hand system.

3.11 SATELLITE COORDINATE DETERMINATION. - - All navigational satellite systems require accurate orbital information so that the satellite coordinates at any point in time may be calculated. Since it is not possible to predict the position of a satellite with respect to time to the required accuracy for more than a day it is impossible to include a satellite ephemeris in a nautical almanac.[3]

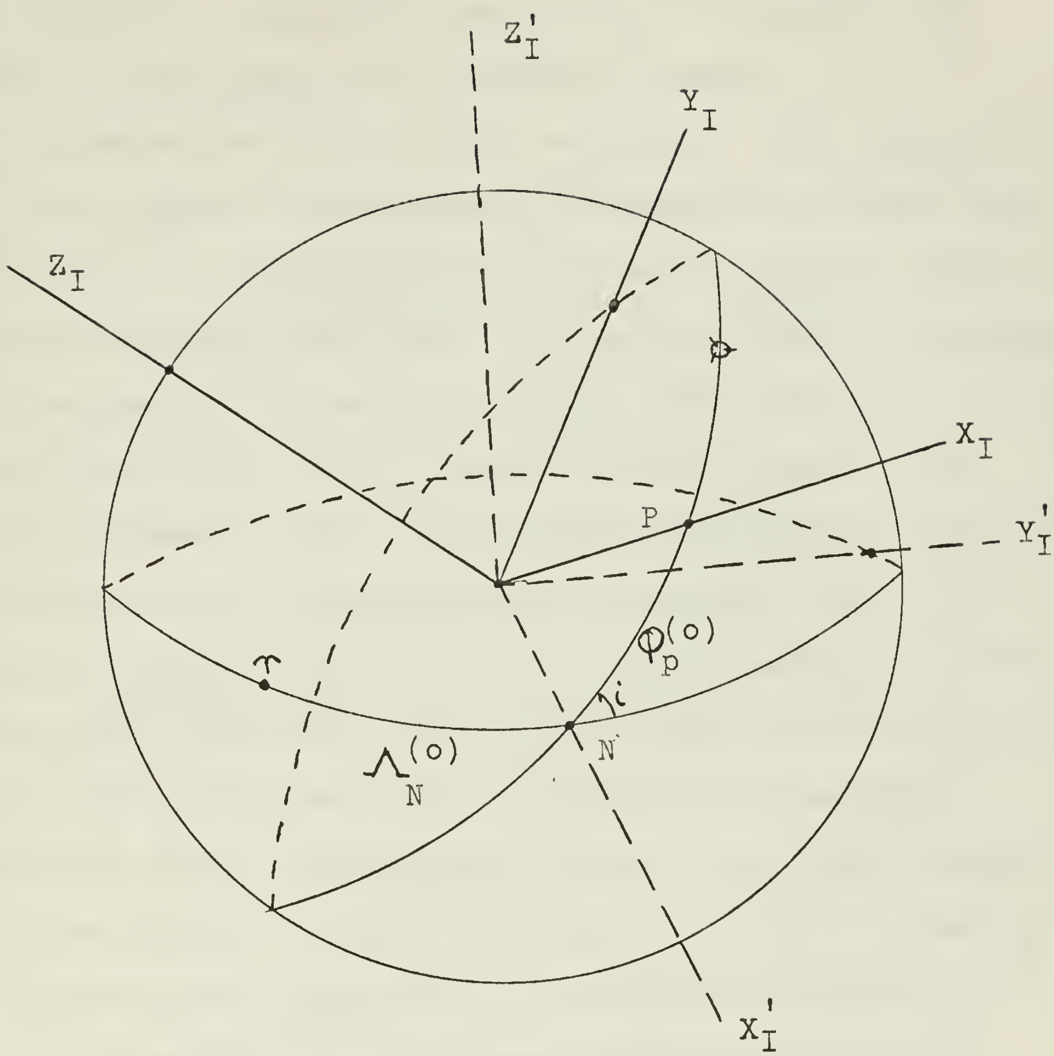


FIGURE 3.5 - INERTIAL COORDINATE SYSTEM

To overcome this drawback the satellite contains an ephemeris table stored in a magnetic memory which is periodically updated with new parameters from the ground.

The memory is divided into two parts. The first or fixed part contains the parameters required to define the best elliptical approximation to the actual orbit. These parameters change slowly with time and are taken as constant over a twelve to sixteen hour period. The descrepancies between this orbit and the actual orbit are small. The second or ephemeral part of the memory contains the small corrections to the basic orbit for each time point. The orbital parameters stored by the satellite memory are given in Table 3.1.[3] From this data the navigator is able to compute the position of the satellite in inertial rectangular coordinates at the beginning and end of each time interval.

To calculate the satellite coordinates X_I^s, Y_I^s, and Z_I^s the navigator goes sequentially through the following calculations where t_m is the mean satellite pass time.[3]

(3.43) $$t_m = t_m - t_p$$

$$\varphi_p = \varphi_p^{(o)} + \omega_p \Delta t_m$$

$$\Lambda_N = \Lambda_N^{(o)} + \omega_N \Delta t_m$$

$$M_m = \underset{360}{\mathrm{MOD}}(h \cdot \Delta M + \omega_o \, \delta t_m$$

TABLE 3.1 - SATELLITE ORBITAL PARAMETERS

Fixed parameters:

t_p = time of first perigee after 0^h or 12^hUT.

ω_o = mean angular motion = 2π/period.

$\varphi_p^{(o)}$ = argument of perigee at t_p.

ω_p = precession rate of perigee.

ε = eccentricity of orbit.

A_o = mean semi-major axis.

$\Omega_N^{(o)}$ = right ascension of ascending node at t_p.

ω_N = precession rate of ascending node.

C_i = $\cos i$ = i = inclination of orbital plane.

S_i = $\sin i$.

ΔM = change of mean anomaly for 1 hour.

δM = change of mean anomaly for 2 minutes.

TABLE 3.1 (Cont)

Ephemeral parameters:

t_i * time after integral half hour E_i, A_i.

δE_i * correction to true anomaly at t_i.

δA_i * correction to semi-major axis at t_i.

$$M_i = M_m + (t_i - t_m)\, \delta M/2$$

$$\sigma_i = \varepsilon \sin M_i$$

$$E_i = M_i + \sigma_i + \delta E_i$$

$$A_i = A_o + \delta A_i$$

Then the satellite coordinates in the inertial system at time t_i are given by:

$$X_I^s = A_i(\cos E_i - \varepsilon)$$

(3.44) $$Y_I^s = A_i \left[\sin E_i (1 - \varepsilon^2)^{\frac{1}{2}}\right]$$

$$Z_I^s = 0$$

These are the satellite coordinates in the defined inertial coordinate system at the i th time point. In order to be used for position determination in (3.18) the inertial coordinates must be transformed to the defined earth rectangular coordinate system.

3.12 TRANSFORMATION OF INERTIAL COORDINATES. - - The transformation of the inertial satellite coordinates to the earth rectangular coordinate system can be considered in two distinct steps. The first step involves two rotations to bring the inertial X_I and Y_I axes into the equatorial

plane and the Z_I axis to coincide with the rotational axis. The results of these rotations define the X_I', Y_I', and Z_I' axes illustrated in Figure 3.5. The second step involves bringing the X_I' axis to coincide with the X axis. This is complicated by the earth's rotation about its axis. Since the earth rectangular coordinate system rotates with respect to the inertial coordinate system the second step takes the form of a rotation of the inertial prime system about the Z axis through an angle of $-\Lambda_N$ - GAST. Upon completion of these steps the satellite coordinates will be in the earth rectangular coordinate system used in the position derivation. The matrix form of the required rotations is given below:

Step 1 - rotation about the Z_I axis through an angle of $-\varphi_p^{(o)}$ followed by a rotation about the X_I' axis through an angle of $-i$.

(3.31)

$$\begin{bmatrix} X_I' \\ Y_I' \\ Z_I' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-i) & \sin(-i) \\ 0 & -\sin(-i) & \cos(-i) \end{bmatrix} \begin{bmatrix} \cos(-\varphi_p^{(o)}) & \sin(-\varphi_p^{(o)}) & 0 \\ -\sin(-\varphi_p^{(o)}) & \cos(-\varphi_p^{(o)}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_I \\ Y_I \\ Z_I \end{bmatrix}$$

Step 2 - rotation about the Z axis through an angle

of $\left(-\Lambda_N - \text{GAST}\right)$.

(3.32)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos(\Lambda_N - \text{GAST}) & -\sin(\Lambda_N - \text{GAST}) & 0 \\ \sin(\Lambda_N - \text{GAST}) & \cos(\Lambda_N - \text{GAST}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_I' \\ Y_I' \\ Z_I' \end{bmatrix}$$

3.13 SUMMARY - - The theoretical development of equations for position determination given in the preceeding sections illustrates how a navigator is able to fix his position on the surface of the ellipsoid used as an assumed figure of the earth from observations of a navigational satellite. Initial assumed coordinates X_n, Y_n, and Z_n are required from some independent source. In addition, the navigator must determine the integrated cycle count from at least three satellite observations and be able to calculate the coordinates of the satellite at the time of these observations. The solution involves the determination of corrections dX_n, dY_n, and dZ_n which are applied to the assumed initial coordinates. This gives the new position coordinates in a rectangular system. These coordinates are then expressed in terms of geodetic latitude and longitude.

4. OPERATIONAL SYSTEM COMPONENTS

4.1 GENERAL. - - The Navy Navigation Satellite System has been operational since July 1964. It consists of three satellites, four tracking stations, an operations/ computer center, two injection facilities, and numerous navigational receiving units. The ground network is based entirely within the United States and is operated by the U. S. Navy Astronautics Group. The locations of the components of the ground network are shown in Figure 4.1.[6]

The task of the Astronautics Group is to determine, process, and load orbital and time information into the memories of the operational satellites. These operations must be carried out within critical time limits. Each satellite is tracked every time it comes within range of one of the four tracking stations. The data received from the satellite is then sent to the computer which updates the orbit, predicts the satellite positions for the next 16 hours, and generates the orbital parameters in a coded message. This message must be sent to the injection facility in sufficient time to be injected into the satellite before it has used up all of the information previously stored in its memory. Each cycle requires about 12 hours per satellite. Since the satellite memory is limited to 16 hours of information, operations must meet an exacting schedule.

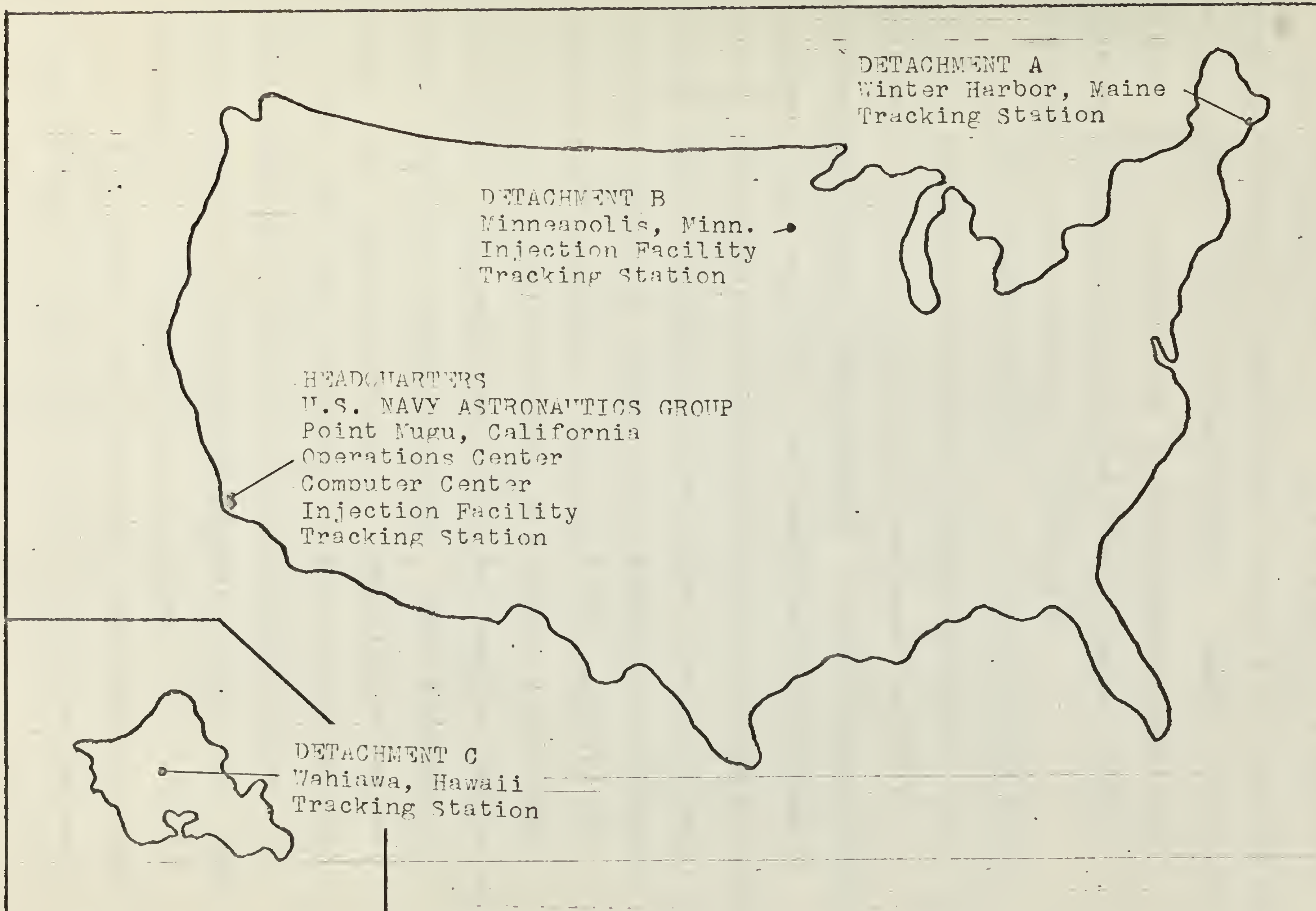


FIGURE 4.1 - LOCATION OF OPERATIONAL GROUND COMPONENTS.

4.2 THE SATELLITE. - - As was previously mentioned, details concerning the operational satellite are not readily available due to security restrictions. However, it can be assumed that the operational satellite employs the basic characteristics developed by the Transit research and development series described in Chapter 2.

4.3 THE TRACKING STATION. - - The four operational tracking stations are located in Maine, Minnesota, California, and Hawaii. The tracking station equipment consists of a Westinghouse AN/BRN-3 type receiver, a doppler cycle counter, an ultra-stable oscillator, and communications equipment. The data obtained by the tracking station is put into standard teletype format consisting of two parts. The first part is called the header and contains the identifying information required by the computer. The second part contains the actual data. The data consists of a series of 11-digit data points. Each point represents one doppler reading and the time that the reading was taken. These readings are usually taken every four seconds. Generally 100 or more data points are obtained during the pass of one satellite within radio line-of-sight of the tracking station. In messages sent to the operations/computer center, the first five digits of a data point represent the time of day in seconds (UT) and the last six digits represent the number of microseconds it takes to receive a predetermined number

of cycles from the satellite. This preselected value is such that the time approaches but never equals one second.

In order to send the great quantity of data gathered by the tracking stations to the operations/computer center, high speed communication equipment is required. A rapid data-link called TRADAT, which has a transmission rate five times faster than teletype, is presently being used. TRADAT is essentially error free because it incorporates self-checking circuitry.

4.4 OPERATIONS/COMPUTER CENTER. - - The operations/computer center is located at the headquarters of the U. S. Navy Astronautics Group at Point Mugu, California. All data from the tracking stations and the computer pass through the operations center. This center at all times maintains the operational system status and the positions of the satellites as well as related systems information. It is the operations center's responsibility to insure that the tracking stations track each satellite and that sufficient data is provided to the computer to allow accurate orbit determinations and predictions. It schedules the computer center runs and insures that the orbital and time parameters are sent to the injection facility in time to meet the injection pass of a satellite.

Four satellites in orbit will produce an average of 84 messages per day from the tracking stations. The data in these messages pass through the operations center and

are sent to the computer center whose primary job is orbit updating and position prediction. The computer center has an IBM 7094 II/1410 configuration with a 360/40 for command and control functions. The doppler data are fed into the computer which is programmed for orbital analysis. The machine computes a theoretical orbit of the satellite and compares a theoretical doppler curve obtained from this orbit with the actual doppler curve. The residuals are squared and summed and a new set of corrected variables is obtained. These allow the computation of a new theoretical orbit and a new theoretical doppler curve to be again compared with the observed data. This process is repeated until the sum of the squares of the residuals is a minimum. At this point it is assumed that the best determination of the orbit has been made. In this way orbits of high precision can be determined and the future position of the satellite can be predicted with the required accuracy. It takes nearly two hours to update a single satellite orbit and prepare an injection message. With each satellite requiring this information approximately every twelve hours, the computer will be used to capacity. The message prepared by the computer contains 24,960 bits of orbital and time information which comprise the message proper and 220,000 bits of information which control the equipment at the injection facility during a satellite pass.

Although TRADAT speed and accuracy are particularly adapted to communications between the tracking stations and the operations/computer center, it still does not provide the speed and accuracy level required to handle the injection messages which are sent from the operations/computer center to the injection facility. These must be 100 per cent error free. High-speed equipment with a data rate more than twice that of TRADAT is used for the sole purpose of sending injection messages, which, from the time they are fed into the equipment, are entirely automated until they are transmitted by the injection facility antenna to the satellite.

4.5 THE INJECTION FACILITY. - - The operational system employs two injection facilities, one in California and one in Minnesota. Each injection facility contains a 60 foot parabolic directional antenna in conjunction with a 10 kilowatt transmitter. A small computer similar to that used in the AN/BRN-3 submarine navigation equipment controls the operation. The injection message from the operations/computer center is received at the injection facility and stored. When the satellite is in range of the injection facility the message proper containing bits of information in combinations of binary ones and zeros is converted to waveform to modulate the carrier wave of the transmitter. The message is transmitted into the satellite

in 15 seconds. Since the satellite and the earth are in motion during transmission, the distance from the transmitter to the satellite varies constantly. Therefore, the time required for the signal to reach the satellite must be computed in advance and the bit rate must be adjusted to ensure that the satellite receives them at a even rate. The bit rate is changed 32 times during the 15 second transmission. The injection facility also reads the message as retransmitted from the satellite to determine the success of the injection attempt. If necessary, the injection facility recycles to do the whole operation over again. A complete cycle for each injection requires two minutes. The equipments of the injection facility consist of commercially available components wherever possible. In fact the transmitter is a modified commercial television transmitter.[8]

4.6 NAVIGATION EQUIPMENT. - - Two types of shipboard navigation equipments have been developed, one for submarines and the other for surface ships. The AN/BRN-3 was developed by Westinghouse and is designed specifically for submarine use. It incorporates many features adapted to submarine environment, and certain pieces of interface equipment to permit operation in conjunction with other navigational equipments presently used aboard Fleet Ballistic Missile submarines. The AN/BRN-3 is completely automatic in operation. It computes it's own alerts of when navigational

satellites will be within radio range of the submarine position. It then computes the satellite position for specific intervals during a pass based upon knowledge it already has of the satellite orbit. Using an assumed submarine position it precomputes the expected doppler curve. When the actual doppler curve is received the AN/BRN-3 adjusts and readjusts the assumed position until the closest possible matching of the two curves is achieved. Since the assumed position is supplied by an Inertial or some other high quality positioning system, the curve-fitting process takes place almost in real time. The position at which the two curves fit most closely together is taken to be the submarine's position. This fix is transmitted to the other navigation equipment on board, as necessary. In addition, the equipment performs its own readiness checks, diagnoses any trouble, and prints out in english what part to replace.

The equipment developed for surface ships is designated the AN/SRN-9. It is far less complex than the submarine equipment since it does not have the environmental and interface problems associated with undersea operations. The AN/SRN-9 receives the signal from the satellite and automatically computes the ship position. It uses the orbital parameters provided by the satellite to compute the part of the orbit corresponding to the doppler curve received. From this it computes the satellite coordinates

at the observation time points. With this information, plus the integrated doppler cycle count for at least three 2 minute time intervals, it computes the corrections dX_n, dY_n, and dZ_n to the assumed position coordinates. The new position coordinates are then used to calculate the navigator latitude and longitude.

5. SYSTEM ACCURACY

5.1 GENERAL. - - In the course of the development of the Navy Navigation Satellite system, individual error components were analyzed in order to determine and improve those components which most affect the system. Theoretical values based on assumed idealized conditions were determined for most components. These values can be expected to vary under operating conditions. Specific values pertaining to the accuracy of fix determinations based on the use of the prototype and operational systems are not available due to security restrictions. However, several fleet units have made extensive use of the operational system and fix accuracies of ± 0.1 mile or better have been made public.

From the navigator's point of view the primary error components of the operational system are:

1. Ionospheric refraction error.
2. Frequency error.
3. Time error.
4. Error in the determination of the velocity of electromagnetic propagation.
5. Navigator instrument error.
6. Navigator velocity error.
7. Satellite coordinate error.

A short discussion of these error components is given in

[illegible]

the following sections. It should be noted that due to the nature of the system, the satellite coordinate error can be broken down into individual error components similar to those encountered in fix determination. The principal difference is in the high order gravity coefficients required for orbit determination and prediction.

5.2 IONOSPHERIC REFRACTION. - - In Section 3.2 it was shown that a first order correction for ionospheric refraction can be determined by the transmission of two coherent frequencies from the satellite. The resulting doppler shift still contains higher order effects of the series ($1/f_s$). In order to determine whether contributions higher than the first order are significant, the theoretical effect of refraction on the doppler shift was considered in Reference 9. Table 5.1 summarizes the results pertaining to upper bounds on the various contributions to the refracted doppler shift for various frequencies within the transmission range.[9] Except for the vacuum term, typical values are probably a factor of three to ten smaller. The first order term is proportional to the time rate of change of the total number of electrons in a unit cross section tube extending from the observation point to the satellite. This is the term that is eliminated when two frequency doppler data is used. The second order term considers Faraday rotation and its sign depends on whether the antenna is left

f_s (mc/s)	Δf_v	$\frac{\Delta f_1}{f_s}$	$\frac{\Delta f_2}{f_s^2}$	$\frac{\Delta f_{3,1}}{f_s^3}$	$\frac{\Delta f_{3,2}}{f_s^3}$	$\frac{\Delta f_{3,3}}{f_s^3}$	$\frac{\Delta f_{3,4}}{f_s^3}$
50	1,200	20.0	.60	1.92	.63	-	-
100	2,500	10.0	.15	.23	.08	-	-
150	3,700	6.7	.07	.04	.02	-	-
200	5,000	5.0	.04	.03	.01	-	-
300	7,500	3.3	.02	.01	-	-	-
400	10,000	2.5	.01	-	-	-	-
500	12,500	2.0	-	-	-	-	-

TABLE 5.1 - ESTIMATED MAXIMUM CONTRIBUTIONS TO REFRACTED DOPPLER SHIFT. (Entries are in cps.)

or right circulary polarizod. The third order correction is composed of four terms. The first term arises from the fourth order term in the expansion of the refractive index series. The second term represents the difference between the optical path and the geometric path. The last two terms represent modifications to the solution of the wave equation using the geometrical optics approximation. These terms depend upon various derivatives of the electron density. Unless the satellite frequencies are below VHF the two terms not obtainable from the geometrical optics approximation are negligible.

Table 5.2 presents the estimated maxima of the second and third order contributions to the vacuum doppler shift when dual frequency data are combined to eliminate the first order refraction term.[9] Three typical frequency combinations are given and the table entries have been converted from frequency shift to errors in the range rate in meters per second.

Examination of the tables indicates that if the lower of the two frequencies is not less than 100 mc/s and the ionosphere is not extremely disturbed, the second and third order refraction contributions should be negligible.

5.3 FREQUENCY AND TIME ERRORS. - - The navigator must have an accurate clock and a reference frequency in order to determine his position from doppler observations.

f_1 (mc/s)	f_2 (mc/s)	2nd Order	3rd Order
50	300	.10	.26
100	300	.05	.01
150	400	.02	.01

TABLE 5.2 ESTIMATED MAXIMUM CONTRIBUTIONS TO DUAL FREQUENCY DOPPLER DATA. (Entries are in meters/sec.)

In addition he must know the frequency transmitted by the satellite and the satellite position as a function of time. In Chapter 3 it was shown that to the required accuracy the frequency of the reference oscillator is assumed unknown and the position determination computations were arranged so as to eliminate the need for this value. The navigator is able to recover time from each satellite observation to an accuracy of better than one millisecond. This time is maintained by a clock derived from the reference oscillator which has a drift rate of less than 1 part in 10^{10} per day. Therefore, the time and frequency errors in the Navy Navigation Satellite System both derive from the drift of the satellite oscillator. A drifting oscillator gives an along track navigation error because the frequency is in error and causes a time error in the clock that it is driving. This time error also gives an along track error. Navigation errors of about 1 kilometer result from an error of 1 part in 10^8 in the satellite frequency. However, satellite frequency can be determined with an accuracy of at least 1 part in 10^{10} and predicted with an accuracy of at least 2 parts in 10^{10} per day. Since the satellite oscillator can be tuned by command from an injection station twice per day, it's frequency can be kept to a predetermined value within an accuracy of a least 2 parts in 10^{10}. This causes a navigation error of approximately 20 meters.[10]

5.4 VELOCITY OF ELECTROMAGNETIC PROPAGATION. - - The value for the velocity of electromagnetic energy propagation is taken to be a constant in calculations for position determination using the Navy Navigation Satellite System. The value is probably known to an accuracy of 1 part in 3,000,000. This would permit position determination to an accuracy of about two meters.[5] Future improvements in this constant can be accomodated by modification of the system based on the current accepted value.

5.5 NAVIGATION INSTRUMENT ERROR. - - In order to test the contribution that the navigation equipment makes to the navigation error, observations were made on experimental satellites transmitting on four frequencies. Two navigation instruments at the same site made simultaneous observations on the satellite. One instrument used two of the satellite frequencies and the other used the remaining two frequencies. The difference between the navigated positions is then the result of two independent measurements of position. The result of one set of tests obtained with the Transit 2A satellite gave a mean difference in position of 50 meters in one coordinate and 30 meters in the other.[3] It is expected that instrumentation has improved since this time.

5.6 NAVIGATOR VELOCITY ERROR. - - Since the navigator must use a measured value of his velocity to determine his position at the times of the satellite observations, an

error in this measurement will cause an error in the fix. A ship acted on by unknown current, windage drift, and sea condition effects amounting to a one knot velocity error in the northerly direction has an estimated position error of 0.3 nautical miles.[12] A one knot easterly velocity error causes a negligible positional error at mid-latitudes since the satellite is in a polar orbit. It was determined to at least three place accuracy that the position errors varied linearly with the velocity errors.[13] This does not appear to be a severe limitation for surface ships and submarines where accurate pit-log or inertial system velocities are normally available.

5.7 SATELLITE COORDINATE ERROR. - - Apart from the preceeding error components which are normally negligible, the only remaining source of system error is in the satellite coordinates transmitted to the user. Any error in these coordinates is reflected almost exactly one for one in the accuracy of the navigation fix. Tests on the size of the errors in the predicted orbits of the Navy Navigation Satellite System were conducted by computing and predicting the orbits at four tracking stations. These predictions were made using geodetic parameters in use at the Naval Weapons Laboratory during four periods of time through 1964. The predicted orbit was compared with an orbit based upon independent doppler observations made by world-wide tracking stations during the prediction interval and using the latest

available gravity coefficients. The resulting twelve hour prediction errors for random sets of observations made on polar orbiting satellites show a decrease in size from over 1000 meters in 1960 to the value of 50 meters in 1965.[11]

5.8 PROPOSED ERROR ANALYSIS. - - A high speed computer may be easily programed to analyze the effect of individual and/or collective error components on a given satellite position determination. The basic inputs of this program would consist of the following parameters:

1. X_n, Y_n, Z_n
2. $i\Delta x$, $i\Delta y$, $i\Delta z$
3. X_i^s, Y_i^s, Z_i^s
4. f_r
5. f_s
6. c
7. ΔT

The satellite coordinates as a function of time in an earth centered rectangular coordinate system are available from a Geodetic Science Computer Library Program at the Ohio State University or from some other similar program. Using satellite coordinates bracketing three or more time intervals and choosing the above parameters which are assumed errorless, we can determine the expected integrated cycle count from (3.7). Assuming errorless instrumental and environmental conditions the calculated cycle counts and the above parameters may be considered as an ideal errorless position determination based on satellite observations. This can be verified by substitution of these values into the equations developed in Chapter 3. The position

coordinates produced by calculation will be equal to the initial navigator coordinates since the corrections dX_n, dY_n, and $d\Delta f_n$ will be zero in our assumed errorless condition. We have thus created an ideal errorless position determination which can be used as a standard in an error analysis. By random and/or systematic alteration of the assumed errorless input parameters we can determine their individual and/or combined effect on the final position determination. A study could also be made to determine the effect of decreasing the time interval and using more observations in a least squares solution.

6. SUMMARY

The adaption of space technology provides a new approach to the earth-bound problem of marine navigation. The Navy Navigation Satellite System is based on the doppler shift in a radio frequency transmitted from a near-earth satellite. By exact measurement of the doppler shift it is possible to determine the location of the receiver on the surface of the assumed figure of the earth. The accuracy obtained by using this technique is possible because the satellite is very precisely restricted to an orbit defined by physical laws of motion and the quantities observed, time and frecuency, can be measured to an accuracy of one part in 10^{10}. Of all the possible satellite paths permitted there is only one which results in a particular curve of doppler shift.

Two basic problems are inherent in this system. One is that the ionosphere bends radio waves and thus gives a false position of the satellite. This has been overcome by the use of two satellite transmission frequencies which allow a first order correction to be computed. The second basic problem is the determination of the gravity field of the earth. This affects the determination and prediction of satellite orbits. This problem has been reduced to an acceptable level by using satellite orbital data to calculate new gravity coefficients.

The use of a navigational satellite system based on

the doppler shift of VHF radio waves appears to provide the best present day solution to the military navigation requirements. World-wide coverage is obtained by use of polar orbits. The system is inherently all weather and immune to interference since line-of-sight radio frequencies are used for both tracking and navigation. Since the satellites transmit their ephemerides the navigator need neither interrogate the satellite nor receive orbital and time information by other communication links. The system is also reliable because a temporary interruption of reception during a pass does not preclude the use of data that are received to obtain an accurate navigational fix. It is assumed that the system meets the military accuracy requirements.

BIBLIOGRAPHY

1. Bowditch, N. : American Practical Navigator. U.S. Navy Hydrographic Office Publication #9, Washington, D.C., 1958.

2. Cawley, J. : Review Of Marine Navigation Systems And Techniques. Arthur D. Little, Inc., Cambridge, Mass., Report No. 1510165, January 1965. AD460836.

3. Kershner, R. B. and Newton, R. R. : The Transit System. The Journal of the Institute of Navigation, Vol. 15, No. 2, London, April 1962. AD614162.

4. Freitag, R. F. : Project Transit. Speech made at the Institute of Navigation, Colorado Springs, Colorado, 23 June 1960.

5. Nicolaides, J. D., Macomber, M. M., and Kaula, W. M. : Terrestial, Lunar, and Planetary Applications of Navigation and Geodetic Satellites. Advances in Space Science and Technology, Vol. 5, Academic Press, New York and London, 1963.

6. Rueger, L. J. : Precision Time and Frequency Applications in the Transit and Anna Satellite Program. Applied Physics Laboratory, The Johns Hopkins University, Report No. TG-459, 14 December 1962. AD416596.

7. Guier, W. H. and Weiffenbach, G. C. : Theoretical Analysis of Doppler Radio Signals from Earth Satellites. Applied Physics Laboratory, The Johns Hopkins University, Report No. 276 (Bumblebee series), April 1958.

8. U. S. Navy Navigation Satellite System. U. S. Navy Astronautics Group, Point Mugu, California, August 1964. (Public Information Booklet)

9. Guier, W. H. : Ionospheric Contributions to the Doppler Shift at VHF from Near-Earth Satellites. Applied Physics Laboratory, The Johns Hopkins University, Report No. CM-1040, July 1963. AD600285.

BIBLIOGRAPHY
(continued)

10. Newton, R. R. : Everyman's Doppler Satellite Navigation System. Applied Physics Laboratory, The Johns Hopkins University, Report No. TG-590, July 1964. AD609208.

11. Anderle, R. J. : Error Analysis of the Navy Doppler Tracking System Applied to Geodetic Operations. U. S. Naval Weapons Laboratory Technical Memorandum No. K-14/65, January 1965. AD455644.

12. Guier, W. H. and Weiffenbach, G. C. : A Satellite Doppler Navigation System. Proceedings of the IRE, 48, 4, April 1960.

13. Myers, H. A. : Aircraft Navigation by Satellite. The Rand Corporation, Santa Monica, California, September 1962. AD605633